OCEANOGRAPHY EEES 2400
EXERCISE 2a – Powers of Ten and Calculations

To start, take a virtual tour of the powers of ten:
http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/

Power of ten is the mathematical shorthand for writing very large or very small numbers, using positive or negative exponents of base 10. The exponent is written as a superscript numeral to the right of the 10.

The Basics:

(1) Ten with a positive exponent means to multiply 10 by itself the number of times indicated by the power. For example, $10^3 = 10 \times 10 \times 10 = 1000$.

(2) Ten to a negative power means to multiply 0.1 (1/10) by itself the number of times indicated by the power. For example, $10^{-3} = 0.1 \times 0.1 \times 0.1 = 0.001$ (equivalent to 1/1000, or the reciprocal of 10^3, which is 10^{-3}).

(3) Ten to the zero power (10^0) equals 1, by definition. Zero (0) can only be represented by itself.

(4) Any number can be expressed as a power of ten by using a *scalar value* and an *exponent*. For example, $212 = 2.12 \times 100 = 2.12 \times 10^2$ and $0.0098 = 9.8 \times 10^{-3}$.

(5) When ***multiplying*** two numbers expressed as powers of ten, add the exponents. For example, $10^3 \times 10^2 = 10^{(3+2)} = 10^5$.

(6) When ***dividing*** two numbers expressed as powers of ten, subtract the exponents. For example, $10^3 / 10^2 = 10^{(3-2)} = 10^1$ and $10^2 / 10^3 = 10^{(2-3)} = 10^{-1}$.

(7) When ***multiplying*** *or* ***dividing*** numbers in scientific notation, multiply (or divide) the scalar values separately from the exponents, then combine the two.
For example, $(1.2 \times 10^3) \times (3.0 \times 10^5) = (1.2 \times 3.0) \times (10^3 \times 10^5) = 3.6 \times 10^8$

$$(6.0 \times 10^3) \div (2.0 \times 10^2) = (6.0 \div 2.0) \times (10^3 \div 10^2) = 3.0 \times 10^1$$

(8) When ***adding*** or ***subtracting*** numbers expressed in powers of ten, transform the values to a common exponent, then add or subtract the scalar (non-exponent) values. For example, $3.8 \times 10^3 - 1.2 \times 10^2 = 3.8 \times 10^3 - 0.12 \times 10^3 = 3.68 \times 10^3$.

To make thinking in large and small numbers a little easier, scientists and engineers commonly express powers of ten in multiples of 3 (or –3), which is equivalent to counting, 1 thousand, 1 million, 1 billion, 1 trillion, and so on. In computer lingo, this idea is expressed by the prefixes kilo-, mega-, giga-, and tera-, as in “megabytes.”

10^X	Equivalent value	10^X	Equivalent value
9	1 000 000 000	–9	0. 000 000 001
6	1 000 000	–6	0. 000 001
3	1 000	–3	0. 001
2	100	–2	0.01
1	10	–1	0.1
0	1	0	1

Other things to keep in mind about math with exponents:

Use a leading zero in the ones place for any decimal values
for example, 0.025 instead of .025

When using scientific notation, express the scalar value with a numeral in the ones place
for example, 1.25×10^5 instead of 0.0125×10^7.

The exception to the above guideline is when you need to use a common exponent for operations such as addition and subtraction;
for example, $(1.25 \times 10^3) + (0.75 \times 10^3) = 2.00 \times 10^3$
instead of $(1.25 \times 10^3) + (7.5 \times 10^2) \neq 8.75 \times 10^3$

Be careful about moving the decimal point and the exponent in the correct direction (especially for negative exponents). In your mind (or on paper), convert the number expressed in scientific notation into a decimal value, and ask yourself whether the exponent gets larger or smaller. (Admittedly, this can be tricky.)

For example,
0.254×10^2 ($0.254 \times 100 = 25.4$) becomes 2.54×10^1, NOT 2.54×10^3 (which is 2540)
and
25.4×10^2 ($25.4 \times 100 = 2540$) becomes 2.54×10^3, NOT 2.54×10^1 (which is 25.4)

Likewise,
0.15 is equivalent to 1.5×10^{-1} or 0.015×10^1 (moving the decimal point either way)

A rule of thumb for decimal places and exponents:

For positive exponents, 1×10^n n is equal to the number of zeroes to the *right* of 1.
Examples: $1 \times 10^3 = 1\ 000$ $1 \times 10^6 = 1\ 000\ 000$

For negative exponents, 1×10^{-n} there are (n-1) zeroes to the *left* of 1
(i.e., the equivalent decimal position would be to the right of the 1)
Examples: $1 \times 10^{-3} = 0.001$ $1 \times 10^{-6} = 0.000\ 001$

Remember the relationship between positive and negative exponents and reciprocals of fractions:

$1 / 10^3 = 1 \times 10^{-3}$ and $1 / 10^{-6} = 10^6$

AND, when adding or subtracting with scientific notation, always remember to convert to a common exponent:

$$\begin{array}{r} 1.25 \times 10^3 \\ +\ \underline{0.75 \times 10^4} \end{array} \quad \text{convert to} \Rightarrow \quad \begin{array}{r} 1.25 \times 10^3 \\ +\ \underline{7.50 \times 10^3} \\ 8.75 \times 10^3 \end{array} \quad \text{which is equivalent to} \Rightarrow \quad \begin{array}{r} 1250 \\ +\ \underline{7500} \\ 8750 \end{array}$$

Name ____________________________

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Express each number as a power of ten:

235	=	____________________	0.00235	=	____________________
186,000	=	____________________	0.000186	=	____________________
3,950,000	=	____________________	0.39500	=	____________________
25 trillion	=	____________________	one billionth	=	____________________
1	=	____________________	0	=	____________________

(can zero be represented as a power of ten?)

Express each as a non-"power of ten" number:

2.35×10^{4}	=	____________________	0.235×10^{-2}	=	____________________
18.6×10^{6}	=	____________________	1.86×10^{-5}	=	____________________
38.50×10^{3}	=	____________________	38.5×10^{-6}	=	____________________
2.77×10^{9}	=	____________________	1×10^{-9}	=	____________________
1×10^{7}	=	____________________	0.01×10^{-4}	=	____________________

Complete each calculation, show intermediate steps when appropriate, and express the answer as a power of ten:

$(5.8 \times 10^{4}) \times (1.7 \times 10^{5}) =$

$(4.4 \times 10^{6}) / (2.4 \times 10^{3}) =$

$(0.36 \times 10^{-3}) / (1.2 \times 10^{5}) =$

$(0.46 \times 10^{-3}) \times (1.2 \times 10^{5}) =$

$5.6 \times 10^{4} + 2.8 \times 10^{5} =$

$6.5 \times 10^{-3} - 6.5 \times 10^{-4} =$

$4.7 \times 10^{22} - 5.3 \times 10^{23} =$

$0.59 \times 10^{2} + 59 \times 10^{-2} =$

$0.012 + (2.2 \times 10^{-1}) \times 10^{0} =$

$(5.4 \times 10^{2}) \times (2.3 \times 10^{-3}) \times (-1) =$